

# MEDICAL IMAGE SEGMENTATION USING GENERALIZED GRADIENT VECTOR FLOW AND CLIFFORD GEOMETRIC ALGEBRA

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**Abstract-** Recently, there have been several researchs done on the aim of computer-aided diagnosis by using medical images. The early warning system based on analysing the medical images able to detect the anomaly on the patient easily. In this paper we discussed a new method for detecting the shape of the object on the medical images by segmentation.

Segmentation was accomplished by applying generalized gradient vector flow (GGVF) and Clifford geometric algebra. The edge of the object would be identified well using GGFV and then smoothing would be done at the edge based on geometric algebra framework for higher dimension. Segmentation resulted using this method was expected to be closed to the object shape.

CT Scan, dental panoramic radiograph, and mammogram have been used as the experimental images. It was shown that segmentation results by using the proposed method seemed to be closed to the original shape of the object.

Index Terms- segmentation, generalized gradient vector flow, Clifford geometric algebra.

## I. INTRODUCTION

Medical image is an image obtained by shooting active sensor likes X-rays to human body so that yielded an image of patient organ. Generally, medical image has several noises likes illumination, low contrast, and noises. Caused by those noises there are some unclearly objects in medical image. To help a doctor in analyzing an anomaly, medical image processing is needed to find out the shape of object in unclearly medical image. One of the methods is by object segmentation on medical image to find out the shape of object in the medical image. This is done to help a doctor in analyzing the anomaly on the patient earlier so that could reduce the death rate.

Strong algorithm is needed in medical image segmentation. Strong in here is needed a system which could do segmentation fastly and accurately. There have been several segmentation methods. One of the widely known segmentation method is Generalized Gradient Vector Flow (GGVF) and Clifford geometric algebra [7]. The method needed initialization for developing a good shape by drawing a circle on the target object. Since the shape of each objects in the medical images relied on the area of interest and varied each other, we need to relaxed the initialization shape. In this paper the circle will be changed to the ellipse which more easily to match and close with the target object.

Generalized Gradient Vector Flow (GGVF) method is used in automatically directing on selection of input image pattern [3] and [5]. Furthermore, a process is done to get better result in Clifford geometric algebra framework. The Clifford geometric algebra is used because it has some advantages such as geometric object transformation likes point, line, flat plane, circle, and sphere could be expressed in clearly shape.

## II. GENERALIZED GRADIENT VECTOR FLOW AND CLIFFORD GEOMETRIC ALGEBRA

Segmentation using generalized gradient vector flow (GGVF) and Clifford geometric algebra give suitable and accurate result. There are two main processes in this method. These processes are process to choose input using GGVF and process in Clifford geometric algebra. GGVF is used to get coordinate position of object edge. To give more accurate result smoothing process is done in higher dimension, three dimension, in clifford geometric algebra. Furthermore, the smoothing result is reverse again in beginning dimension, two dimension, so that the output of segmentation process can be seen as noticed object shape.

### 2.1. Generalized Gradient Vector Flow

The first step of segmentation process is by selecting the pattern of input image automatically. In here generalized gradient vector flow (GGVF) is used [3], [5], and [7]. GGVF is dense vector field that token from image by minimizing the energy function. The minimizing energy function process is done by measuring a pair of partial differential equation which spreads gradient vector from edge map of gray-level image that measured from image. The energy in here are two kinds, there are internal power which comes from curve in deformation process and external power which comes from the image it self. A gray-level image,  $I(x,y)$ , is appeared as sequential position function from  $(x,y)$  variable. The external power from image is as follows :

$$E_{ext}^{(1)}(x, y) = -|\nabla I(x, y)|^2, \quad (1)$$

$$E_{ext}^{(2)}(x, y) = -|\nabla(G_\sigma(x, y) * I(x, y))|^2, \quad (2)$$

where  $G_\sigma(x, y)$  is two dimensional Gaussian function with standard deviation  $\sigma$  and  $\nabla$  as gradient operator. If the image is line (black on white), than the accurate external power are:

$$E_{ext}^{(3)}(x, y) = I(x, y), \quad (3)$$

$$E_{ext}^{(4)}(x, y) = G_{\sigma}(x, y) * I(x, y). \quad (4)$$

To find the GGVF field, start from determining an edge map,  $f(x, y)$ , which taken from image  $I(x, y)$  that has bigger property close to image edge. It could be used gray-level and binary edge map that has been determined in image processing literature, such as:

$$f(x, y) = -E_{ext}^{(i)}(x, y), \quad (5)$$

where  $E_{ext}^{(i)}(x, y)$ ,  $i = 1, 2, 3$ , and  $4$ , are external power that has been said in equation (1), (2), (3), and (4).

This paper using edge map equation (5) with value  $i = 2$ , so that the equation becomes:

$$f(x, y) = -|G_{\sigma}(x, y) * I(x, y)|^2. \quad (6)$$

By edge map is resulted from equation (6), the GGVF is defined as a vector field  $v(x, y) = [u(x, y), v(x, y)]$  which minimizing energy function :

$$\varepsilon = \iint g(|\nabla f|) \nabla^2 v - h(|\nabla f|)(v - \nabla f). \quad (7)$$

And could be written as :

$$v_t = g(|\nabla f|) \nabla^2 v - h(|\nabla f|)(v - \nabla f), \quad (8)$$

where :

$$g(|\nabla f|) = e^{-\left(\frac{|\nabla f|}{k}\right)^2}, \quad (9)$$

$$h(|\nabla f|) = 1 - g(|\nabla f|), \quad (10)$$

and  $k$  is koefisien that defines how wide target ratio between the smooth region and the suitable gradient. Example of dense vector field is in Figure 1.

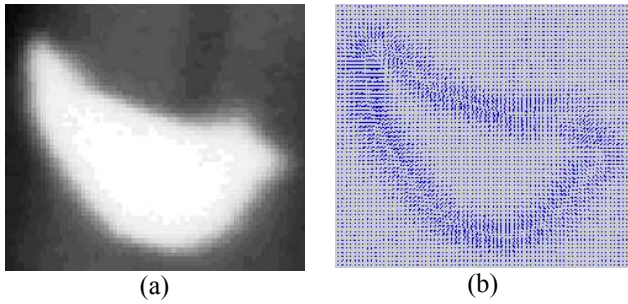


Figure 1 Example of vector field that called GGVF field. (a) Original image ; (b) vector field as a result of GGVF in original image.

After GGVF field value is known, the next process is GGVF field normalization in order to give more clearly boundaries that signs confluence peak of vector flows which shows edge. Then, deformation process is done to identify the object's edge in noticed image. Deformation process is done by using a curve initialed before in object. Deformation process at a curve  $x(s) = [x(s), y(s)]$ ,  $s \in [0, 1]$ , can apply equation :

$$E = \int_0^1 \frac{1}{2} (\alpha |x'(s)|^2 + \beta |x''(s)|^2) + E_{ext}(x(s)) ds. \quad (11)$$

Where  $\alpha$  and  $\beta$  are weight parameter that manages elasticity and rigidity in succession. Whereas  $x'(s)$  and  $x''(s)$  are the first and the second differential of  $x(s)$  to  $s$ . Elasticity dan rigidity

are elements of internal strength that derived from inside curve. External Energy,  $E_{ext}$ , is function of image property to get small value in image feature desired, like a boundary. In this research,  $E_{ext}$  is normalized GGVF field value.

## 2.2. Clifford Geometric Algebra

Clifford Geometric Algebra method is introduced by William Kingdon Clifford (1845-1879) in his paper titled "On The Classification of Geometric Algebras". He conscious that eksterior algebra which found by Grassmann and quaternion algebra which found by Hamilton could be held in a same algebra with a little transition in their exterior product. By the new product which now called geometric product, product in quaternion rules will follow directly the vector base combination, where the Grassmann eksterior algebra unabandoned [2].

Clifford Geometric Algebra has two important element i.e: Clifford Algebra and Geometric Algebra [2]. Clifford algebra discusses about algebra aspect and geometric algebra discusses about geometric aspect. In geometric algebra discussion, will be discussed about 3-D Euclidean vector space notated with  $E^3$ . The coordinate used to represent  $E^3$  is  $R^3$ .

Have to be known that some notational aspect,  $R^{p,q}$ , show a vector space with dimension  $n = p + q$ . An orthonormal base on vector space notated by  $\{e_1, \dots, e_n\}$  and has  $p$  elements with positive sign and  $q$  elements with negative sign. Positive and negative signs are important and needed when the scalar product is defined in vector space. The negative sign means that the quadrat of orthonormal vector base is minus one (-1). For example, a Minkowski space which usually notated by  $R^{n+1,1}$  which placed in  $n + 2$  dimension, where the Minkowski space has orthonormal base which consists of  $n + 1$  vector bases, in which if it quadrated the result is +1 and a vector base, in which if it quadrated the result is -1.

In  $E^3$  space, there are three orthonormal vector bases usually notated with  $e_1, e_2$ , and  $e_3$ . In geometric algebra, there are some basic products like scalar product, outer product, inner product, and geometric product.

Geometric algebra has an outer product, usually called with wedge product. Outer product has anti-symmetry, linearity, and asosiativity properties. There are some outer product result such as bivector and trivector. Bivector is the result of outer product in two kinds of vector. It such a field that has direction. Trivector is the result of outer product in three kinds of vector. It is a volume that has direction. Blade is an object which can be written as an outer product of vectors. As an example,  $e_1$ , or  $e_1 \wedge (e_1 + 2e_2)$ , or  $I^3 \equiv e_1 \wedge e_2 \wedge e_3$ . Grade of blade is subspace dimension where the blade represented. For example in blade  $e_1 \wedge (e_1 + 2e_2 + 2e_3)$ , the grade is 2 (because  $e_1 \wedge e_1 = 0$ ). Another example is blade  $I^3 \equiv e_1 \wedge e_2 \wedge e_3$ , the grade is 3.

Besides outer product, inner product is the important operation in geometric algebra. In Euclidean vector space, inner product, called dot product, is usually used for measuring length and degree. In  $G_{3,0}$  geometric algebra, inner product from two vectors is equals with inner product from two vectors in Euclidean space. In vectors, inner product of geometric algebra has standard properties, symmetry and linearity.

Inner product and outer product show perpendicularity and parallelity, but both of them not give perfect relationship between two vectors. Hence, there is a new product operation from the combination of two products operation, inner product and outer product. The new product operation is called geometric product. This Geometric product makes Clifford algebra different with others. A geometric product equation is as follows :

$$ab = a \cdot b + a \wedge b, \quad (12)$$

where a and b are two vectors in  $E^n$  space. Equation 12 shows that the geometric product has two parts, there are scalar part  $a \cdot b$  and bivector part  $a \wedge b$ .

Each of inner product and outer product determines uncompletely relationship among vectors, hence inner product and outer product cannot be inverted. However geometric product, combination of inner product and outer product, has invertible objective. This is the most important thing in geometric algebra.

In geometric algebra, there is a subspace where an object of the geometric exist. The subspaces are point, line, plane, circle, and sphere. Usually most people believe that ‘point’ equal with ‘vector’, the really vector is direction measurement and not a point. However point and vector have very close relationship. Point can be represented with a vector in “1 more dimension”. This is called homogeneous model [9]. The rule base of homogeneous model is to draw n dimensional space using off-set subspace so that the delineation is placed in (n+1) dimensional space. Method for laying in (n+1) dimensional space is by expanding the space using an addition vector “e” perpendicular with space n. For example a point in space n usually represented with:

$$x = a_1e_1 + a_2e_2 + \dots + a_n e_n \quad (13)$$

In homogeneous model, a point P in space n which reside in p position, is represented by a vector in space (n+1) as follows :

$$P = e + p. \quad (14)$$

### 2.3. Segmentation



Figure 2 Block diagram method is used in this paper.

Generally, segmentation method used in this paper is shown in figure 2. In the figure 2, can be seen that the first step

in segmentation is measuring GGVF vector field and determining the shape of image edge with deformation. Then the next step is maping the result which consist of koordinat position of boundary object to the three dimensional spaces ( $G_{3,0}$ ), and than smoothing is done by adding a new point between two nearly points in the three dimensional spaces. The smoothing result is reversed again in two dimensional spaces by mapping again from three dimensional spaces ( $G_{3,0}$ ) to two dimensional spaces in order to the result could be seen. Another purposes is in order the result could be compared with segmentation result in original image.

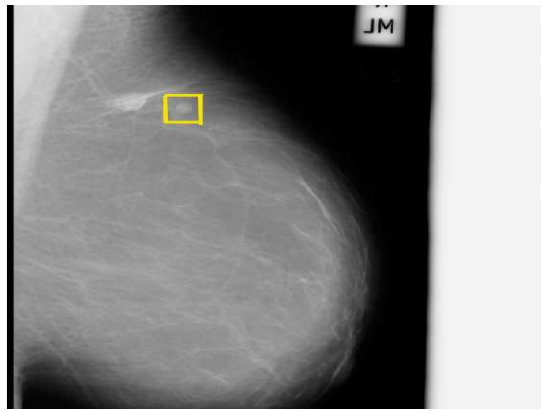
An image used in segmentation process in this method is region of interest (ROI) containing an object that will be segmented. The ROI is taken from gray-scale image in medical image of original image.

### III. RESULT AND DISCUSSION

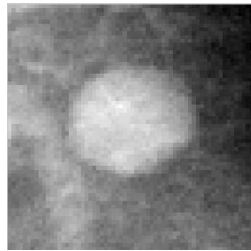
Figure 3 and figure 4 show the result of this method if this method is used in mammogram images containing breast cancer, to get shape of micro calcification in mammogram. The difference are in figure 3 uses circle as the deformation curve initialization and in figure 4 uses ellipse as the deformation curve initialization. Figure (3a) and figure (4a) show original mammogram image and the ROI (Region of interest) of object that will be segmented, there is breast cancer. Figure (3b) and figure (4b) show ROI from original image that will be processed. Figure (3c) and figure (4c) shows edge map of ROI image. Figure (3d) and figure (4d) are normalization of GGVF field. Figure (3e) and figure (4e) are deformation curve initialization. Figure (3f) and figure (4f) are mapping result from two dimensional spaces to the three dimensional spaces ( $G_{3,0}$ ) and the smoothing result. Figure (3g) and figure (4g) show ROI image and segmentation result with red and yellow lines.

Segmentation using GGVF and Clifford geometric algebra method in medical image like panoramic radiograph is shown in figure 5 and figure 6. The difference are in figure 5 uses circle as the deformation curve initialization and in figure 6 uses ellipse as the deformation curve initialization. Figure (5a) and figure (6a) show panoramic radiograph image and ROI (Region of interest) from object that will be segmented. Figure (5b) and figure (6b) show ROI regions from panoramic radiograph image that will be processed. Figure (5c) and figure (6c) show edge map from ROI image. Figure (5d) and figure (6d) are normalization of GGVF field. Figure (5e) and figure (6e) are deformation curve initialization. Figure (5f) and figure (6f) are mapping result from two dimensional spaces to the three dimensional spaces ( $G_{3,0}$ ) and the smoothing result. Figure (5g) and figure (6g) show image ROI and segmentation result with red and yellow lines. From all experiments, red and yellow lines in output image are segmentation result using GGVF and Clifford Geometric algebra. By comparing input image and output image can be seen that in figure (3) and figure (5), the segmentation result is like the object noticed. However in figure (4), the segmentation result is not like the object noticed. This is because there are noises such as neurons near the object so that influence the deformation process. By numeric measuring, succession rate of figure (3) is 97,84 %, in figure (4) is 96,91 %, and in figure (5) is 99,15 %.

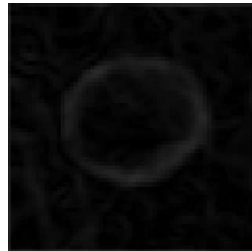




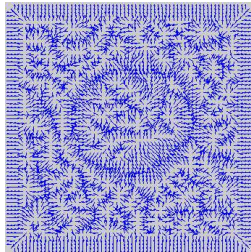
(a)



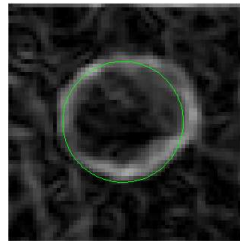
(b)



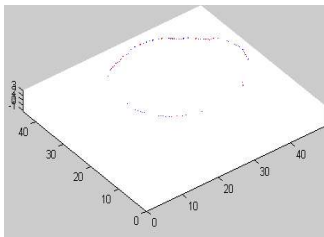
(c)



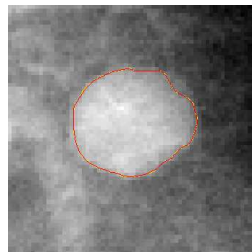
(d)



(e)



(f)

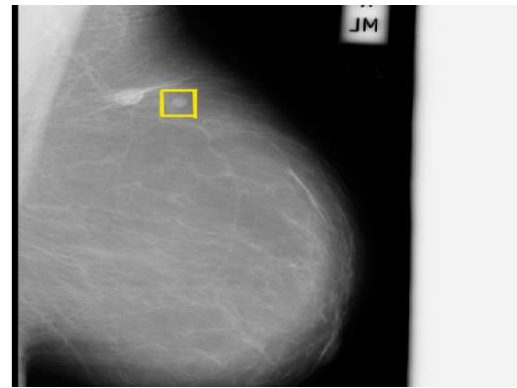


(g)

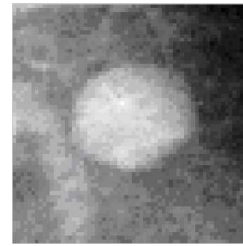
Figure 3 Experiment of segmentation in mammogram image that containing breast cancer with circle as the initial curve. (a) mammogram image with breast cancer in ROI; (b) ROI image; (c) edge map; (d) GGVF field normalization; (e) initial curve; (f) mapping in three dimensional space; (g) segmentation product in ROI image.

#### IV. CONCLUSION

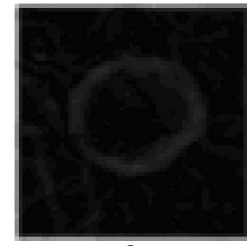
Boundary of the target objects as the result of the experiments has been well shown using generalized gradient vector flow and Clifford geometric algebra, either using circle or ellipse initialization shape.



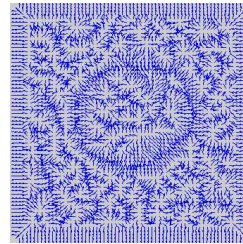
(a)



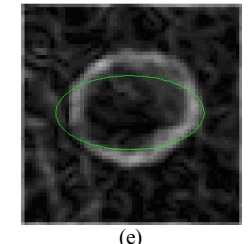
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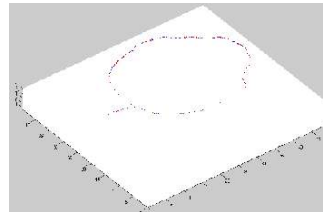
(c)



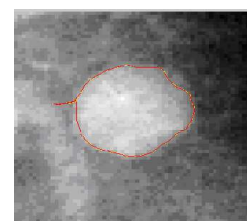
(d)



(e)



(f)



(g)

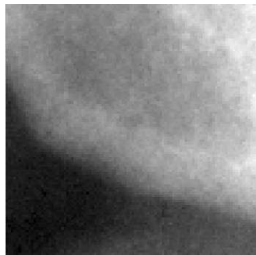
Figure 4 Experiment of segmentation in mammogram image that containing breast cancer with ellipse as the initial curve (a) mammogram image with breast cancer in ROI; (b) ROI image; (c) edge map; (d) GGVF field normalization; (e) initial curve; (f) mapping in three dimensional spaces; (g) segmentation product in ROI image.

Generalized gradient vector flow is used, because calculation using GGVF reduces energy function in input image so that the system will be more powerful and faster. The smoothing process used higher dimension, because in the higher dimension there are many empty spaces between nearly coordinate where if in dimension two the empty space is not seen. In this paper such process was done in three dimensions, by adding a point between two nearly point.

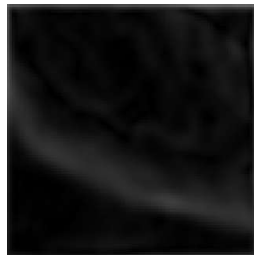
Result of segmentations using generalized gradient vector flow and Clifford geometric algebra have been similar with the object.



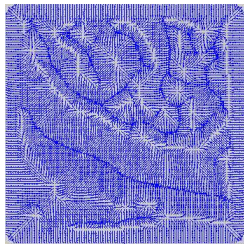
(a)



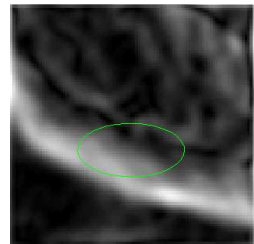
(b)



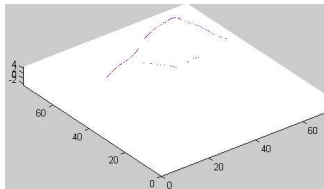
(c)



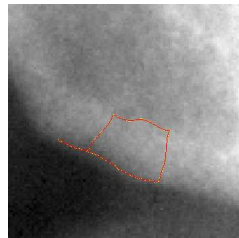
(d)



(e)



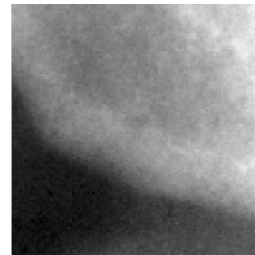
(f)



(g)



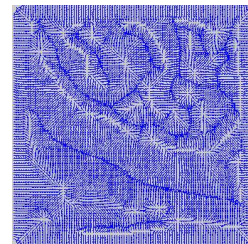
(a)



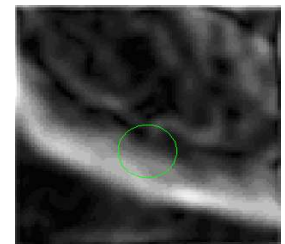
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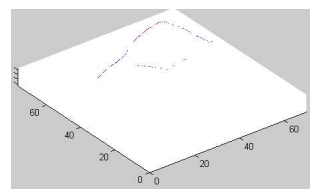
(c)



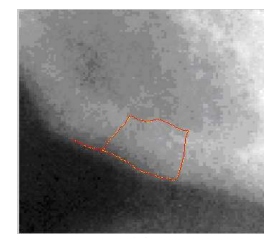
(d)



(e)



(f)



(g)

Figure 5 Experiment of segmentation in panoramic radiograph image with ellipse as the initial curve. (a) panoramic radiographs with ROI region; (b) ROI region; (c) edge map; (d) GGVF field normalization; (e) initial curve; (f) mapping in three dimension space; (g) segmentation product in ROI image.

Figure 6 Experiment of segmentation in panoramic radiograph image with circle as the initial curve. (a) panoramic radiographs with ROI region; (b) ROI region; (c) edge map; (d) GGVF field normalization; (e) initial curve; (f) mapping in three dimension space; (g) segmentation product in ROI image.

## V. REFERENCES

- [1] J. A. Kanis, O. Johnell, Requirements for DXA for the management of osteoporosis in Europe, *Osteoporos Int*, 16, 2005, pp. 229-38.
- [2] A. Taguchi, M. Tsuda, M. Ohtsuka, I. Kodama, M. Sanada, T. Nakamoto, K. Inagaki, T. Noguchi, Y. Kudo, Y. Sueti, K. Tanimoto, A. M. Bollen, Use of dental panoramic radiographs in identifying younger postmenopausal women with osteoporosis, *Osteoporosis International*, 17, 2006, pp. 387-394.
- [3] Agus Zainal Arifin, A. Asano, A. Taguchi, T. Nakamoto, M. Ohtsuka, M. Tsuda, Y. Kudo, and K. Tanimoto, "Computer-aided system for measuring the mandibular cortical width on dental panoramic radiographs in identifying postmenopausal women with low bone mineral density," *Osteoporosis International*, 17, 5, 2006, pp. 753-759.
- [4] T. Nakamoto, A. Taguchi, A. Asano, M. Ohtsuka, Y. Sueti, M. Fujita, M. Sanada, K. Ohama, K. Tanimoto, Computer-aided diagnosis of low skeletal bone mass on panoramic radiographs, *J Dent Res* 83, Special issue A, 2004.
- [5] F. Yan, H. Zhang, C.R. Kube, A Multistage Adaptive Thresholding Method. *Pattern Recognition Letters* 26, 2004, pp. 1183-1191.
- [6] K. Thangavel dan M. Karman. Computer Aided Diagnosis in Digital Mammogram: Detection of Microcalcifications by Meta Heuristic Algorithms. *International Journal on Graphics Vision and Image Processing*, 7, 2005.
- [7] C. Tomasi and R. Manduchi, Bilateral Filtering for Gray and Color Images. *IEEE International Conference on Computer Vision '98*, 1998, pp. 836-846.
- [8] Nobuyuki Otsu, A Threshold Selection Method from Gray Level Histograms. *IEEE Trans. Systems Man Cybernet.* 9, 1979, pp. 62-66.