IMAGE THRESHOLDING BY MEASURING THE FUZZY SETS SIMILARITY

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ABSTRACT

Finding an optimal gray level threshold that can separate objects from background is an important issue. This paper proposes a new method for thresholding the image histogram into two crisp subsets, based on a fuzzy logic and maximization of a criterion function. In order to evaluate the performance of the proposed method, we apply the proposed method to the poorly illuminated OCR source images. The comparison of the thresholded images by the proposed method with the Otsu's method and Kwon's method, illustrates the effectiveness of this method.

1. INTRODUCTION

Thresholding is a widely used tool in image segmentation for partitioning pixels in a digital image into two nonoverlapping sets. Finding an optimal gray level threshold that can separate objects from background is an important issue. Since image thresholding is a well-researched field, there exist many algorithms for determining an optimal threshold of the image. Sezgin and Sankur have conducted an exhaustive survey of image thresholding methods and categorized them according to the exploited information, such as histogram shape, measurement space clustering, entropy, object attributes, spatial correlation, and local gray level surface [1].

This paper proposes a thresholding method on the basis of histogram image, because it is a computationally efficient way. Usually, such methods imply thresholding a histogram according to the maximization or minimization of criterion function. Several histogram-based approaches have been proposed to segment images, but none is universal and this equation still remains an open problem.

Such an algorithm is the well-known Otsu's algorithm that evaluated the goodness of each gray level if used as the threshold [2]. This evaluation considered the discriminant analysis which maximizes the separability of the resultant classes in gray level. This method selecting a gray level as the threshold, so that the measure of the class separability defined by a criterion function has the maximum value, at which the mean of two classes would be as well-separated as possible and the variances would be as small as possible.

Kittler and Illingworth have proposed the minimum error threshold selection based on the assumption that the object and background gray levels is normally distributed [3]. The principal idea is to optimize a criterion function determined from the average gray levels classification error rate. It is found to give excellent results even in the presence of a wide range of region scatters. In this method, the cost function is derived based on Bayes rule.

These measurements are applied directly to the histograms, which make them computationally efficient. However, if the assumption of similar class sizes is not met, they are found to be unstable [4]. It may be seen that many of these algorithm perform excellently well for a set of images depending on the underlying assumptions and yield poor results often in other situations.

Fuzzy logic, which has an ability to deal with ambiguous data, is a powerful tool to solve the ambiguity or fuzziness caused by the overlapping of the two class probability densities [5]. During the past decade, several fuzzy model based methods have been developed to overcome this difficulty [6]. It has been applied for thresholding image by minimizing the measure of fuzziness of the image [7].

Huang and Wang have developed a fuzzy entropy-based thresholding algorithm, which measure the fuzziness directly in the image histogram [8]. This method utilizes two fuzziness measurements (i.e. Shannon's function and Yager's measurement) for determining the fuzziness of each pixel. The image is viewed as a fuzzy set that reflecting the membership of each pixel to which it belongs. This membership value is calculated based on the absolute difference between the gray level of the pixel and the mean of the region to which it belongs.

We propose a new method for thresholding the image histogram based on a fuzzy logic and maximizing a criterion function. However, differently from previous methods, we do not regard an image as a single fuzzy set. Instead, the proposed method regards an image as two fuzzy sets correspond to the background and objects classes.

To this end, the criterion function represents the similarity measure between both fuzzy sets for evaluating the goodness of each gray level if used as the threshold. Hence, in the proposed method, we do not use any measurement of fuzziness of each gray level to a class for selecting the optimal threshold. Instead, the fuzzy sets similarity of background and objects will conclude the optimal threshold gray level.

2. HISTOGRAM THRESHOLDING

In the implementation of the histogram thresholding algorithm on a basis of fuzzy sets similarity degree comparation, we shall assume:

1. A given histogram image consists of two fuzzy sets corresponded to the black and white regions which are separated by a gray level alone.
2. The membership degree of gray levels located exactly at the center of black region or the left hand side achieve the maximum value for the black fuzzy set and the minimum value for the white fuzzy set.
3. The membership degree of gray levels located exactly at the center of white region or the right hand side of it achieved the maximum value for the white fuzzy set and the minimum value for the black fuzzy set.

The aim of this algorithm is to threshold the gray level histogram into two crisp subsets, using fuzzy set similarity measurement.

2.1. THE FUZZY SET

The fuzzy set concept offers a convenient framework for representing imprecise concepts. A fuzzy membership function denotes the degree of membership of a pixel to a particular class. Let X denote an image set of size $M \times N$, whose pixels L gray levels, in the range $[0, L-1]$ and x_{mn} is the gray level of a (m, n) pixel in X. Let us consider the membership grade by a mathematical function $\mu_A(x_{mn})$ [7].

A fuzzy set A in X is formally denoted as

$$A=\{(x_{mn}, \mu_A(x_{mn}))\}, \quad (1)$$

where $0 \le \mu_A(x_{mn}) \le 1$ and A is characterized by the function $\mu_A(x_{mn})$, which assigned to each pixel.

In this paper, based on the two regions determined in the assumption 1), let us define two linguistic variables {Object, Background}, denoted as fuzzy sets O and B, respectively. We modeled the characteristic functions of $\mu_O(x_{mn})$ and $\mu_B(x_{mn})$ that correspond to the membership function of object and background, respectively. These functions assign to each pixel in the fuzzy set O and B of membership grade in the interval [0, 1]. Given T as the threshold, such membership functions are defined as

$$\mu_O(x_{mn}) = \begin{cases} 1, & x_{mn} \le v_O \\ 1 - \{(x_{mn} - v_O)/D\}, & v_O < x_{mn} < v_B \\ 0, & x_{mn} \ge v_B \end{cases} \quad (2)$$

$$\mu_B(x_{mn}) = \begin{cases} 0, & x_{mn} \le v_O \\ 1 - \{(v_B - x_{mn})/D\}, & v_O < x_{mn} < v_B \\ 1, & x_{mn} \ge v_B \end{cases} \quad (3)$$

where v_O and v_B are the mean gray level value of pixels in object with interval $[0, T]$ and background with interval $[T+1, L-1]$, respectively. They are defined using the histogram information as

$$v_O = \frac{\sum_{z=0}^{T} z.h(z)}{\sum_{z=0}^{T} h(z)} \qquad \text{and} \qquad (4)$$

$$v_B = \frac{\sum_{z=T+1}^{L-1} z.h(z)}{\sum_{z=T+1}^{L-1} h(z)} \qquad (5)$$

where $h(z)$ is the height of the histogram at gray level z. Since the of the proposed thresholding heavily rely on the histogram, for simplicity, we will normalize those height of the histogram.

The parameter D is chosen as $D = v_B - v_O$, i.e. the length of fuzzy region at which each gray level has a membership degree to each group. Note that we assign a highest membership value to background class for all pixels whose gray level less than the center of the background region. This concludes that the calculating membership value of such pixels to the other class is no longer needed, because the gray level of v_B, which larger from them has been assigned as zero membership value to the object class.

We apply the same concept to the gray levels larger than v_O, so that those gray levels will have full membership to the object class and no membership to the background class. In addition, the definition of v_B and v_O indicate that the position of the T is in between or $v_O \leq T < v_B$.

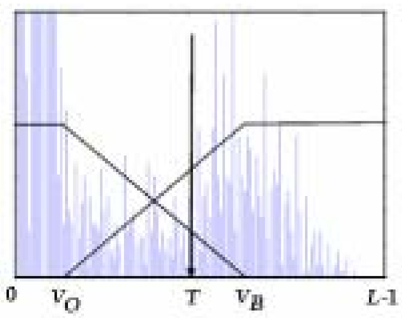


Figure 1. Histogram of a sample image and fuzzy sets *O* and *B* corresponding to gray level *T* as the threshold.

Given gray level T as the threshold, the shape of the membership functions for fuzzy set O, which has the cluster center in v_O and fuzzy set B which has the cluster center in v_B is shown in Fig. 1. Note that gray levels in $[0, v_O]$ and $[v_B, L-1]$ have full membership value and zero membership value, respectively, to the fuzzy set O. While gray levels between v_O dan v_B tend to decrease from one until zero. The fuzzy set B takes the same characteristic into account.

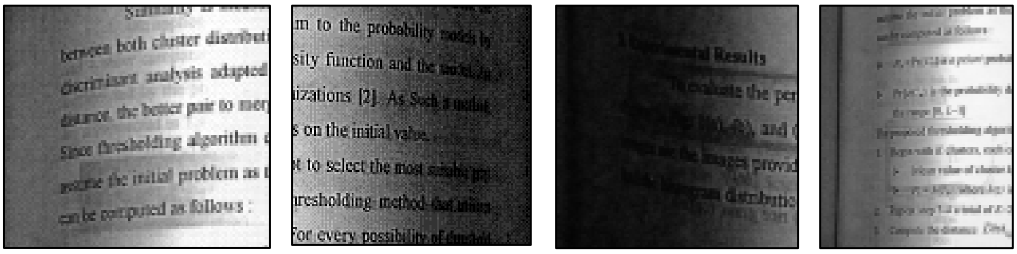
2.2. FUZZY SET SIMILARITY

Distinguishing similar groups or classifying similar elements needs a tool for deciding the similarity degree among them. Similarity measurement of two fuzzy sets A dan B, indicates the degree of similarity of two fuzzy sets. The idea of finding the similarity of such fuzzy sets is regarding the goodness of separability of the two fuzzy sets resulted, when we used a gray level as the threshold value. The lower the measured similarity value, the larger the separability of the two fuzzy sets.

The center of the object region is used as a seed for starting the similarity measure process. Since all gray levels less than this value considered as the member of the object region, we do not need to measure their similarity to the other fuzzy set, because their membership to the other fuzzy set should be zero. Also, we define the center of the background region as a seed for ending the similarity measure process as depicted in Fig. 1.

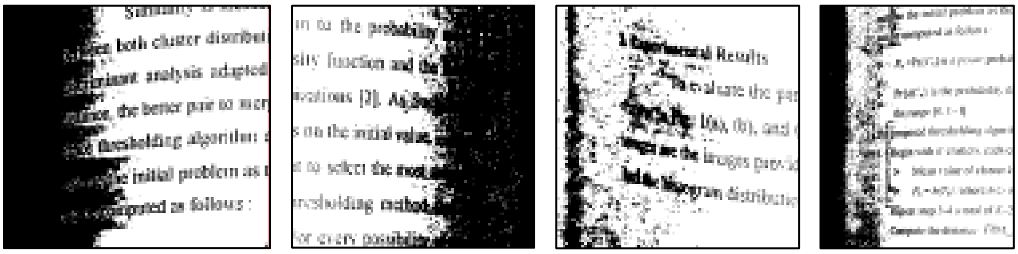
Since all gray levels larger than this value are considered as the member of objects region, we do not need to consider their membership grade to the other fuzzy set. To this end, a similarity measure of two fuzzy sets is used for assessing the separability of the resulted fuzzy sets when we choose a gray level as the threshold.

We assign gray level x_i of the two fuzzy regions, which existed between v_B and v_O, membership values $\mu_B(x_i)$ and $\mu_O(x_i)$ to the fuzzy sets B and O, respectively. The more the difference of membership values of x_i to the fuzzy sets B and O, the better separability of the two fuzzy sets. Hence, for the two fuzzy sets B and O, at a particular gray level x_i, we measure the information amount for discrimination of $\mu_B(x_i)$ to $\mu_O(x_i)$ [9]. In this paper, the information amount for discrimination at gray level x_i $Disc(T, x_i)$, which used T as the threshold value, is defined as



(a) (b) (c) (d)

Figure 2. The original images.



(a) (b) (c) (d)

Figure 3. The thresholded images by the proposed method

$$Disc(T, x_i) = e^{|\mu_O(x_i) - \mu_B(x_i)|}. \quad (6)$$

The reason for calculating from the absolute value is only to consider the difference membership degree of each gray level to the two fuzzy sets. It is obvious that the least difference for gray level at which its membership to the two fuzzy sets has the same value is achieved in the intersection point between two lines reflecting the membership value in the fuzzy region.

2.3. CRITERION FUNCTION

The difference accumulation of membership degree from all gray levels when we choose T as the threshold will indicate the goodness of T in discriminating the two fuzzy sets. Given gray level T as the threshold, the criterion function is defined as

$$J(T) = \frac{1}{M.N} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} e^{|\mu_O(z_{mn}) - \mu_B(z_{mn})|} \quad (7)$$

where M and N are the image size. If all pixels having the same gray levels in Eq. (7), are grouped by using the histogram information, then the equation can be simplified as follows

$$J(T) = \sum_{z=0}^{L-1} Disc(T, z).h(z) \quad (8)$$

where $h(z)$ is the height of the histogram at gray level z and L is the number of bin in the histogram. The optimal threshold is chosen to maximize $J(T)$, that is

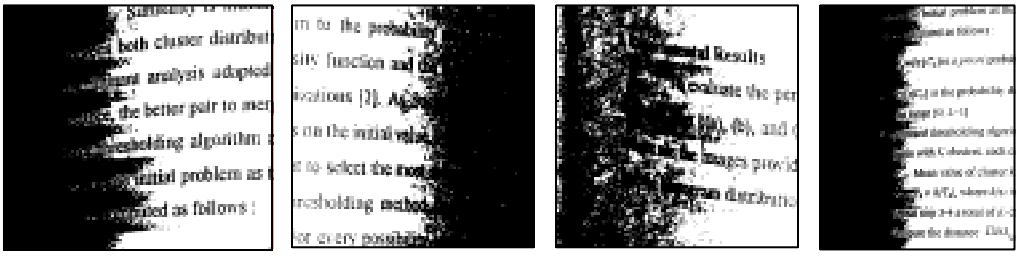
$$T^* = \arg \max_{0 \le T \le L-1} J(T) \quad (9)$$

where T^* denote the optimal threshold at which the discrimination of the two fuzzy sets is maximized.

3. EXPERIMENTAL RESULTS

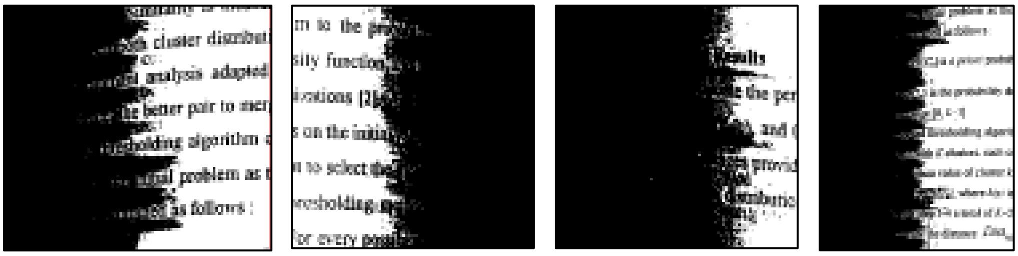
In order to evaluate the performance of the proposed method, our algorithm has been tested experimentally. We apply the proposed method to images 1, 2, 3, and 4, as shown in Fig. 2(a)–(d), respectively. These images are poorly illuminated OCR source images, which contained noises caused by inappropriate way of scanning and the appearance of some texts printed on the back page. Since all images have gray levels ranged from 0 to 255, we will select the most suitable one used as the threshold among 256 gray levels based on Eq. (9).

In addition to the proposed method, two other methods, which are Otsu's thresholding method [2] and Kwon's threshold selection method [10] are used for comparison. The reason for choosing the two methods is that they have similar basis of



(a) (b) (c) (d)

Figure 4. The thresholded images using Otsu's method



(a) (b) (c) (d)

Figure 5. The thresholded images using Kwon's method

selection in which the criterion function is used for evaluating the goodness of a gray level as the threshold.

The thresholded images using the proposed method for images 1, 2, 3, and 4, respectively are shown in Figs. 3(a)–(d). The wide noise regions caused by the different illumination at the left hand side of image 1, 3, and 4 or the right hand side of image 2 cannot be removed well. However most of all texts can be seen, even from the image with very dark background as shown in Fig. 3(c).

Table 1. Threshold values for the experimental images

Method	**Image**			
	1	**2**	**3**	**4**
P	123	54	12	93
O	164	96	57	159
K	140	60	19	128

Note: P, O, and K are the proposed method, Otsus's method, and Kwon's method, respectively.

Figs. 4 and 5 illustrated the thresholded images obtained using Otsu's method and Kwon's method, respectively. The images shown in Figs. 4(a)–(d) and 5(a)–(d) correspondence with those shown in Figs. 3(a)–(d). Some texts seem to be covered by the background, especially those, which are poorly illuminated and closed to the dark area. The threshold values determined for each image using the three different methods are summarized in **Table 1**.

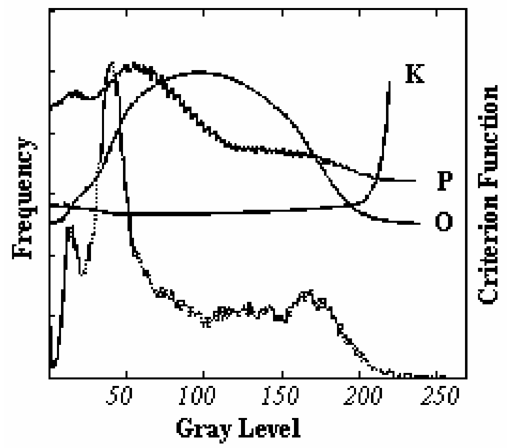


Figure 6. The curves of the criterion function obtained by the proposed method (P), Otsu's method (O), and Kwon's method (K).

Fig. 6 shows the curves corresponding to the criterion function of the proposed method, Otsu's method, and Kwon's method, which are denoted as P, O, and K, when they were applied to the histogram of the image 2. The threshold values of the proposed method, Otsu's method, and Kwon's method are 54, 96, 60, respectively. Note that the proposed method and the Otsu's method select their threshold value by maximizing the criterion function, while Kwon's method by minimizing it.

The curve of the proposed method tends to be drastically changed at the area around the maximum gray level, which is selected as the threshold. While the curves of the two methods seem to be flat at the corresponding area. This condition suggests the robustness of the proposed method, regarding to the noise sensitivity.

4. CONCLUSION

In this paper, we have presented a new gray level thresholding algorithm based on the similarity of two obtained fuzzy sets, when we used a gray level as the threshold value on a histogram. This algorithm minimized the similarity of the fuzzy sets to get maximum separability of the two regions.

The fuzzy framework is used to obtain the membership degree of each gray level to the objects and background sets. The optimal threshold value is selected at a gray level, at which the difference accumulation between membership degrees to the two fuzzy sets is maximal. This concept attempted to achieve non-overlapping regions of the histogram, which contents the objects and the background regions.

The comparison of the thresholded images by the proposed method with the Otsu"s method and Kwon's method, illustrate the effectiveness of the proposed method.

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